Rising damp: capillary rise dynamics in walls

By Christopher Hall1,* and William D. Hoff2

1School of Engineering and Electronics, and Centre for Materials Science and Engineering, The University of Edinburgh, Edinburgh EH9 3JL, UK
2School of Mechanical, Aerospace and Civil Engineering, The University of Manchester, Manchester M60 1QD, UK

We analyse rising damp using the concepts and methods of unsaturated flow theory. A simple first-order Sharp Front model is developed which uses clear physical principles and includes the effects of evaporation and gravity. We find that the simple model captures well the observed features of capillary rise in walls and is supported by the underpinning nonlinear capillary diffusion theory. For most cases, capillary forces are dominant and the effects of gravity can be neglected.

Keywords: capillary rise; evaporation; sorptivity; clay brick; building stone

1. Introduction

Rising damp is the common term for the slow upward movement of moisture in the lower parts of walls and other ground-supported structures. It is an important cause of wetness in buildings. It is a cause of decay and deterioration in standing stones, monuments and at archaeological sites. Much has been written about rising damp, some informed and some less so. Experienced practitioners in building conservation have described its features—see, for example, the book by Massari & Massari (1993) and the technical pamphlet from the Society for the Protection of Ancient Buildings (Thomas et al. 1992). Almost all of these accounts are descriptive and qualitative. There is no harm in that: rising damp is a complicated process. What we wish to do in this short paper is to offer a physical analysis of rising damp from which we develop a quantitative model to complement the descriptive accounts. This provides a clear identification of the principal factors which control rising damp, expressed in some simple formulae. These formulae can be evaluated numerically for many cases to provide practical guidance on such matters as the height of rise, the time-scale for drying and so on. We emphasize that these formulae do not represent the phenomenon exactly, although they are the exact results of the simplified model. We show that their predictions are consistent with practical observations. Some features of this analysis are to be found in the paper of Vos (1971) who described the suction of groundwater by walls from the standpoint of a soil physicist.

* Author for correspondence (christopher.hall@ed.ac.uk).
2. Unsaturated flow

(a) Capillary absorption

Our understanding of water transport in building materials (brick, stone and concrete, plaster and mortars) now has a strong scientific basis. The research literature from, say, the 1960s is extensive. We have set out the present state of scientific knowledge in our recent monograph (Hall & Hoff 2002). There are many complications and much remains to be studied in more detail, but we do now understand that the primary physical processes arise from capillary forces at work within the pores of the materials. These forces are responsible for the initial uptake of water from external sources, such as ground water, driving rain or leaks, or through condensation. Capillarity is also the cause of migration within the fabric, the redistribution of water from place to place which is associated usually with the local differences of water content. Eventually, water may leave the structure and the only important means to do this is by evaporation: liquid water turning into vapour. The liquid–vapour phase change may occur at a building surface or inside the fabric to be followed by vapour migration within the material before eventual entry into the atmosphere. These various processes all fall within the scientific theory of unsaturated flow, the term emphasizing that the materials in the building fabric are rarely fully saturated. If they were, capillary forces would be absent and water movement could only occur in response to the external forces: in practice, hydrostatic heads and gravity. As it is, the opposite is more or less the case: almost always in building elements and structures, the capillary forces are dominant.

The description of unsaturated flows can now be represented mathematically with reasonable rigour. The mathematical models are often complicated and frequently can be applied to particular cases only by means of computer-based numerical methods. As a result, water transport modelling of building structures linked to site survey and measurement is almost non-existent in building conservation practice. But it is now technically possible and of course is undertaken increasingly in the design analysis of heat, mass and air transfer for new buildings (Adan et al. 2004). Hamilton (2006) carried out a brief but important exploratory analysis of water flux in a standing stone at the Skara Brae archaeological site using numerical code originally developed for modelling unsaturated flow in hydrology.

Fortunately, there is often a half-way house, where we can use simpler models, which express the essential first-order features of the processes. This allows us to avoid the use of numerical computation and highlights the important factors and their physical interrelationships. There is a trade-off: we obtain insight and understanding, and reasonable estimates of the quantities we calculate, but less detail than we obtain with numerical methods. In the field of unsaturated flow, an important class of simplified models is based on sharp front (SF) theory. Here, the important simplification is to ignore the rather fuzzy boundary between wet and dry regions within a structure or fabric and to replace this by a notional sharp boundary. We then ask how this boundary moves in particular materials as water is fed into the structure from an external supply and perhaps removed elsewhere by evaporation. These SF models are particularly good for dealing with...
geometrical complications and with composite structures where two or more different materials may be present. In this spirit, we now describe an SF model of rising damp.

3. Rising damp

A brief outline of this model first appeared in a footnote in Hall & Hoff (2002). Here, we expand the model and derive a number of new results. We consider the situation represented in figure 1a: we have a structure in hydraulic contact with the ground and of unlimited height. The structure is of constant thickness, $b$, and composed of a porous material. For this analysis, the only material property of the structure that we need to know is its sorptivity $S$. This property is easily measured (Hall 1989). If the structure is composed of several materials, we need to know only some kind of averaged or composite sorptivity, as we discuss later.

We take as our starting point the proposition that rising damp is the result of competition between the capillary absorption of water along the boundary AA$'$ and the evaporation of water along the exposed surface(s) BB$'$ (Vos 1971; I’Anson & Hoff 1986). We denote the total rate of absorption along AA$'$ by $U$ and the total evaporation rate on BB$'$ as $E$. When the height of rising damp $h$ has stabilized at some value $h_{ss}$, we have a steady state in which $U = E$. This is not a state of static equilibrium in which nothing happens, but a dynamic state in which ‘water-in’ is balanced by ‘water-out’. In stabilized rising damp, there is a
steady flow of water through the system \( F_{ss} = E_{ss} = \epsilon h_{ss} \), where \( \epsilon \) is the evaporation rate per unit area of the wetted surface. In fact, the magnitude of the steady flow \( F_{ss} \) is one of the most interesting results of our analysis.

We note one further relation which we use later. The total quantity of water stored within unit length of the wall is \( Q = \theta_w bh \). Here, \( \theta_w \) is the moisture content of the wetted region of the wall. To be precise, it is the volume of water per unit volume of material, averaged over the entire wetted region. The quantity \( \theta_w \) appears in all the rising damp formulae we derive, so its meaning and measurement are important. It may be obtained by direct measurement of the moisture content in the wall. However, we know from many previous research studies that the value of \( \theta_w \) always lies in the fairly narrow range between the volume fraction porosity of the material \( f \) (the conventional ‘porosity’) and the so-called capillary moisture content of the material (the value obtained in a short-term capillary rise experiment). Since the capillary moisture content is rarely less than approximately \( 0.7f \), it follows that to take \( \theta_w = 0.85f \) will not introduce much error. If site measurements are available, so much the better.

Figure 1a is a two-dimensional physical summary sketch of rising damp in a generic wall. Since our aim is to understand the scaling relations in the dynamics of rising damp, we now represent this in the lumped one-dimensional form shown in figure 1b. In this representation, the rate of evaporative loss \( E \) depends only on the height of the wetted region \( h \); whether evaporation occurs on one or both sides of the physical wall is of no interest. The evaporation at every level is lumped, as is the total capillary absorption at the foot of the wall.

We consider the two quantities \( U \) and \( E \) in turn. The water entering the structure along AA’ depends on the capillary water absorption properties of the wall material(s). For almost every construction material, capillary water absorption into a bar of dry material obeys a simple physical law \( i = St^{1/2} \), where \( i \) is the cumulative volume of water absorbed (per unit area of inflow surface) and \( t \) is the elapsed time. This in fact provides a definition of the sorptivity property: \( S \) is easily measured by a laboratory test on a small sample of material. We emphasize here that the sorptivity is not an empirical parameter, but is rigorously defined in the theory of unsaturated flow and the capillary diffusion theory based on the Buckingham–Richards equations (Hall & Hoff 2002).

We note that as the water rises in the wall, gravity exerts a downward force. This effect can easily be included in our model, but at the price of some mathematical complication. For completeness, we include a full SF model with gravity effects in appendix A. However, the capillary forces are generally dominant in walls and we therefore develop the theory here omitting gravitational forces. We compare the results with and without gravity effects later in the paper.

Therefore, now neglecting the role of gravity, we write

\[
U = \frac{b}{2t} \frac{di}{dt} = \frac{1}{2} bS t^{-1/2} = \frac{bS^2}{2t} \quad (3.1)
\]

to describe the capillary absorption into unit length of the wall through AA’.

Since \( i = \theta_w h \), we have

\[
U = \frac{bS^2}{2\theta_w h} \quad (3.2)
\]
Equation (3.2) shows that the rate $U$ at which water is absorbed at the base of the wall varies inversely with the height of rise $h$.

Now we turn to the evaporation component of the model. Here, we make use of a well-established result (van Brakel 1980; Hall et al. 1984) that the rate of evaporation of water from moist porous materials is determined solely by the environmental conditions over a wide range of water contents from saturated to fairly dry. What we mean by fairly dry varies somewhat from material to material, but for example in the case of brick the range of constant evaporation extends from the saturation water content $\theta_s$ to approximately 0.3 $\theta_s$ (Hall et al. 1984; Massari & Massari 1993). We therefore do not need to know much about the damp material below the rising wet front except its extent. However, we do need to have some measure of the drying capacity of the local microenvironment. For this, we use the potential evaporation (the evaporation rate of a free water surface located at the surface BB'): this quantity we call $e$. (We know (Hall et al. 1984) that $e$ is influenced by temperature, air humidity and air flow speed at and close to the wall surface, although the interrelation of these quantities is complex. Both air flow mapping and humidity measurement are difficult, so the ideal site measurement is a direct measurement of $e$. This measurement is rarely if ever made, and it is clear from this and other recent related work that it is a high technical priority.) We then set the evaporation rate per unit area of the wetted surface $e_Z e$.

The evaporation component of the model is thus extremely simple

$$E = e h. \quad \text{(3.3)}$$

The total rate of evaporation $E$ depends on the wetted height $h$ and the evaporation rate (per unit area) $e$ established by the microenvironment.

(a) Steady-state height of rise

We first consider the situation where rising damp has stabilized. Water absorption and evaporative loss are in balance. We then put $U_{ss} = E_{ss}$, where the subscript $ss$ denotes the rising damp steady state. From equations (3.2) and (3.3), we have

$$\frac{b S^2}{2 \theta_w h_{ss}} = e h_{ss}, \quad \text{(3.4)}$$

so that

$$h_{ss} = S \left( \frac{b}{2 e \theta_w} \right)^{1/2}. \quad \text{(3.5)}$$

This simple but important formula tells us that the steady-state height of rise varies as the square root of the wall thickness and inversely as the square root of the potential evaporation rate in the local microenvironment. The height of rise varies also in direct proportion to the sorptivity of the wall material. The functional dependence of $h_{ss}$ on $b$ and $S$ agrees with the earlier analysis of Vos (1971).

There are three important quantities that we can now calculate. First, at steady state, the total quantity of water per unit length of wall $Q_{ss} = \theta_w b h_{ss}$. Second, the steady flow of water through the wall $F_{ss} = e h_{ss}$. From these two

quantities, we see also that the mean residence time of water in the wall is $Q/F$; this is the mean journey time for a water molecule to travel through the wall.

**(b) Putting in some numbers**

Let us consider an illustrative example: a solid masonry wall constructed of building limestone. The sorptivities $S$ of limestones usually lie somewhere in the range $0.5$–$1.5\ \text{mm/min}^{1/2}$, so we take $S=1.0\ \text{mm/min}^{1/2}$. The volume fraction porosity $f$ is generally approximately $0.25$, so we put $\theta_w=0.2$. We set the wall thickness at, say, $150\ \text{mm}$. For the evaporation rate, we take a value of $e=0.001\ \text{mm/min}$, which corresponds to the UK annual potential evaporation averaged over an entire year. If we insert these values into equation (3.5), we obtain a steady-state height of rise of $0.61\ \text{m}$. This value is similar to the heights observed in house walls showing rising damp (Building Research Establishment 1989).

The total water stored in the wall is approximately $18\ \text{L/m length}$ of wall. The total flow through the wall is approximately $0.88\ \text{L/day/m length}$. The total flow is striking, amounting to some $320\ \text{L/year/m length}$. It is unsurprising that we commonly see severe deterioration of the wall fabric, as well as finishes and decorations such as wall paintings. The mean residence time is approximately $21\ \text{days}$.

Equation (3.5) tells us that if we decrease the evaporation rate, the steady height of rise increases, and that $h_{ss} \propto 1/\sqrt{e}$. So let us recalculate the results with a value of $e$ four times smaller than the previous one. We now have a height of rise of $1.2\ \text{m}$; the volume of stored water has increased correspondingly to $37\ \text{L}$. The total flow through the wall however has fallen considerably, from $0.88$ to $0.44\ \text{L/day/m length}$. This is the consequence of doubling the area from which evaporation is occurring (owing to the increased height of rise) but reducing the evaporation rate (per unit area) by a factor of 4. The residence time is now much longer, approximately $84\ \text{days}$.

We can also see the effect of changing the wall thickness. Equation (3.5) shows us that the height of rise $h_{ss} \propto \sqrt{b}$. If we double the wall thickness $b$ from $150$ to $300\ \text{mm}$, we find that the height of rise increases from $0.61$ to $0.87\ \text{m}$.

**(c) Changing the height of rise: the response time-scale**

We now consider the case where the absorption inflow and the evaporation loss are not in balance. Then, the difference between the quantities $U$ and $E$ causes the total quantity of stored water to change, such that $dQ/dt = U - E$. Since $Q=\theta_wbh$, we obtain the differential equation

$$\frac{dh}{dt} = \frac{S^2}{2\theta_w^2} \frac{1}{h} - \frac{eh}{b\theta_w}. \quad (3.6)$$

Equation (3.6) determines how the height of rise varies with time when capillary rise has not stabilized.

We may write this equation more simply as

$$h \frac{dh}{dt} = -ah^2 + c, \quad (3.7)$$

where $a = e/\theta_w b$ and $c = S^2/2\theta_w^2$. 

We solve this equation for the initial value \( h=0 \) at \( t=0 \) and obtain the equation for the entire capillary rise process \( h(t) \)

\[
h^2 = \frac{c}{a} [1 - \exp(-2at)]. \tag{3.8}
\]

At long times as \( t \to \infty \), \( h \to h_{ss} = (c/a)^{1/2} = S(b/2c\theta_w)^{1/2} \), as in equation (3.5). From an initial dry state, the system reaches \( h=0.95h_{ss} \) in a time \( t_{95} = 3b\theta_w/2c \), which we take as the time-scale for reaching rising damp steady state. With \( b=150 \text{ mm}, \theta_w=0.2 \) and \( c=0.001 \text{ mm min}^{-1} \) as before, we find that this time is approximately 31 days. This time-scale is consistent with the observations of Taylor (1998) in a laboratory test on a narrow rectangular block of Lépine limestone 630 mm high, in which the wet front climbed to the upper surface in approximately 17 days in the absence of evaporation (Hall & Hoff 2002). The time-scale \( t_{95} \) is inversely proportional to the evaporation rate, and in poorly ventilated situations may be much longer.

It is common building practice to apply a low-permeability render coat to damp-affected walls. Taken alone, this is a poor treatment for rising damp, since it greatly reduces water loss from the wall and drives the steady-state height of rise upwards. At first, it may appear to be effective because the adjustment in \( h_{ss} \) can be slow and it may take a long time for rising damp to reappear at the top of the render coat. Consider the case of a 215 mm thick masonry wall with an effective composite sorptivity of 0.3 mm min\(^{-1/2}\) and with \( \theta_w=0.2 \). Before rendering, we take \( \epsilon=e \) as usual and set \( \epsilon=1 \times 10^{-4} \text{ mm min}^{-1} \), typical of a poorly ventilated space. The steady-state height of rise is 695 mm. If we now apply a render coat to the wall to a height of 1.25 m, we thereby reduce the water loss from the wall surface and we can represent this in our model as a reduction in the evaporation rate \( \epsilon \). Thus, if we set \( \epsilon=0.1 \epsilon=1 \times 10^{-5} \text{ mm min}^{-1} \) after rendering, damp now rises once again up the wall and re-stabilizes well above the top of the render coat. We calculate from equation (3.8) that the wet front takes approximately 425 days to climb from the initial steady level of 695 mm to reappear above the top of the render coat.

(d) Comparison with practical observations

It is often noted that it is difficult to replicate rising damp in the laboratory. The reasons for this include difficulties in producing a suitable mortar which is sufficiently sorptive. In older walls, it is likely that mortars become more sorptive as a result of the prolonged passage of water through them over long periods of time. Fresh mortars, particularly those containing cements, act as a barrier to rising damp. (We note here the work of Mamillan & Bouineau (1976) on test walls of limestone masonry, which shows that the rising damp halts at the first joint although the properties of the stone would suggest much greater heights of rise.)

When we look at published survey data on older buildings, the predictions of our model are satisfactory. Thus, there is the common observation (Building Research Establishment 1989) that in UK houses built without a damp-proof course, rising damp forms a band of saturated wall typically to a height of 0.5–1 m. This agrees with the predictions for evaporation rates in the range 0.25–1.0 \( \times 10^{-3} \text{ mm min}^{-1} \), which are appropriate for UK conditions.
Massari & Massari (1993) have reported their observations of capillary rise in buildings in Rome, noting the importance of wall thickness and evaporating geometry on the height of capillary rise. From visual surveys of walls and pillars, they define the quantity $h_{ss}/b$ which they call the climb index. They report from their long experience that this quantity lies in fairly narrow ranges for walls and pillars exposed to interior or exterior conditions. This is tantamount to proposing a simple scaling of $h_{ss} \propto b$. Thus, a free-standing square pillar with evaporation from all the four sides has a climb index $h_{ss}/b \approx 1$. Our model (assuming $S=1 \text{ mm min}^{-1/2}$, $\theta_w = 0.2$ and setting $e = 4 \times 0.0016 \text{ mm min}^{-1}$) gives $h_{ss}/b \approx 20/\sqrt{b}$, giving a climb index in the range 1.15–0.90 for pillars of thickness 300–500 mm. Here, we set $e = 854 \text{ mm}$, the annual potential evaporation for Rome (Müller 1983). Similarly, for walls, Massari & Massari (1993) suggest that $h_{ss}$ lies in the range 1.5–5, while our model gives $h_{ss}$ in the range 1–2.8 for typical parameter values. Of course, uncertainties in sorptivity and porosity will affect these results but the important conclusion is general. Our model provides an explanation of the dependence of the height of rise on wall thickness and gives numerical results in good overall agreement with the Massaris’ field observations.

These authors also report that in the Church of San Bernardo in Rome, the exceptional wall thickness of 4 m results in rising damp reaching a height of 5.3 m. This is consistent with our model for $S=1 \text{ mm min}^{-1/2}$ and an average evaporation rate $e$ of $1.8 \times 10^{-4} \text{ mm min}^{-1}$ for each side of the wall, not unreasonable for a brick wall coated with painted stucco.

For experimental support, we might also look to other systems in which water moves slowly through capillary structures at rates controlled by evaporation. Thus, the vascular transport of water in plants has been the subject of scaling analysis (notably West et al. 1999). Here, the dynamics of fluid flow and evaporation limit the height of trees just as water absorption and evaporation limit the height of rising damp. However, the architecture of plant transport systems, in which tapered capillaries have evolved to provide constant hydrodynamic resistance along flow paths to leaves at different heights, is profoundly different from that of walls. Nonetheless, it is striking that the heights of the tallest trees are similar to the maximum heights to which some capillary water is calculated to rise in walls in the absence of evaporation (Gummerson et al. 1980).

4. Gravity effects in rising damp

We have stated that in most situations, capillary forces are dominant and that gravitational drainage plays a minor role. Since we have exact equations for the approach to the capillary rise steady state with and without gravity effects (appendix A), we can support this assertion by calculation. Figure 2 shows a comparison of capillary rise kinetics for two examples in which the steady-state heights of rise neglecting gravitational drainage are 500 and 1000 mm. For these particular values of the parameters $S$, $b$ and $\theta_w$, gravity reduces these steady-state heights to approximately 488 and 951 mm, respectively, on the assumption that the ultimate equilibrium height of rise in the absence of evaporation is 10 m. The effect on the rate of rise is minor. In more extreme cases, where conditions are such that rising damp reaches
greater heights, the influence may be greater. However, such conditions are rare and probably only of practical significance on long time-scales where the evaporation is unusually poor.

5. Model walls and real walls

We recognize that real walls and ground-supported structures are more individual and complicated than the generic model we have drawn in figure 1. Our formulae are exact for the model, but the model will not be an exact representation of a particular building structure. Even so, the model embodies sound descriptions of physical processes and these processes occur in real structures. The model therefore captures essential features of these real structures.

What are the main points of difference between real walls and the schematic walls of figure 1? We comment in turn on geometry, materials and water transfer physics.

(a) Geometry: where is the foot of the wall?

In our model (figure 1), we measure the height of rise from a datum \( h = 0 \) which we set as the lowest level at which evaporation occurs, that is, at the point B, the lowest level at which the wall surface is exposed. The justification for this is that any part of the wall that lies below the surface is likely to be at or close to saturation, although the location of the \( h = 0 \) datum benchmark may vary slightly according to circumstances.

(b) Materials: setting the sorptivity

The sorptivity is the only transport property of the wall material which appears in the model. This property has been measured for many constructional...
materials, including brick, stone, mortars and plasters. However in masonry walls, the effective sorptivity may be lower than the sorptivity of the masonry units because joints and interfaces resist flow. Some guidance on the choice of an appropriate value for the composite sorptivity comes from detailed studies of the capillary absorption into layered composite materials (Wilson et al. 1995; Hall et al. 1996; Hall & Hoff 2002). On the other hand, there is some compensation from another important long-term phenomenon, which was first demonstrated experimentally by Gummerson (Gummerson et al. 1980). Accurate weight measurements showed that steady capillary flow through a single clay brick with one header face in contact with water and all other faces exposed to evaporation led over the course of 2 years to complete saturation. All the air trapped during capillary absorption into the initially dry brick was slowly lost by molecular diffusion. Such full saturation has also been observed in detailed measurements of water content profiles in brick and cement-based materials using nuclear magnetic resonance imaging and X-ray attenuation techniques (Roels et al. 2004; Hall 2007). It is therefore probable that in walls subject to long-term rising damp, the water content of the wetted region will slowly approach full saturation. Consequently, we may expect that both \( \theta_w \) and \( S \) determined in short-term tests underestimate the true values. Thus, \( \theta_w \) may be closer to the open porosity \( f \) than the value 0.85\( f \) which we have used; and the sorptivity \( S \) may be as much as 50% higher than the standard value. Some guidance on this is provided by the recent measurement of the sorptivity under vacuum conditions where air trapping is eliminated (Victoria Pugsley 2006, personal communication).

\( \left( c \right) \) The effect of salts

Our model describes a mass balance in which, at steady state, the rate at which water enters by capillary absorption equals the rate at which water leaves by evaporation. If the water passing through the structure contains appreciable amounts of dissolved salts, then there is an additional component to the mass balance, and there is an accumulation of salts within the wetted region. This leads to a progressive increase in the dissolved salt concentration of the stored water and ultimately may lead to the deposition of solid salt either within the fabric or at the surface. These processes are not incorporated in the model, although some of their effects can be accommodated. The sorptivity can be adjusted if need be by a small change in the value of the viscosity. More importantly, in the presence of salts, the evaporation rate \( \epsilon \) is reduced by a factor \( (p_s - p_0)/(p_1 - p_0) \), where \( p_s \) is the vapour pressure of the salt solution at the surface \( BB' \); \( p_1 \) the vapour pressure of pure water; and \( p_0 \) is the water vapour pressure of the environment. The depression of vapour pressure by salt solutions can evidently have a strong influence on the evaporation rate \( \epsilon \). Indeed, \( \epsilon \to 0 \) as \( p_s \to p_0 \) or \( 100p_s/p_0 \to RH \), the relative humidity of the microenvironment.

6. Relation between SF model and diffusion model

The SF model is a simple representation of the full nonlinear diffusion model of unsaturated flow built on the Buckingham–Richards equation. In order to apply the diffusion model, we need to have more comprehensive information on the transport properties of the wall materials. If we can neglect gravity effects, then
we need to have the full capillary diffusivity property \( D(\theta) \), where \( \theta \) is the water content; if we cannot neglect gravity effects, then we need both the hydraulic conductivity property, \( K(\theta) \), and the hydraulic capillary potential property, \( \Psi(\theta) \) (or alternatively \( D \) and one of \( K \) or \( \Psi \)). Then, with a suitable numerical scheme, the entire two-dimensional moisture distribution field may be calculated as a function of time for particular initial conditions. An example of this calculation is shown in Hall & Hoff (2002).

7. Some conclusions

Our analysis goes beyond previous qualitative descriptions of rising damp and provides a first-order model from which important scaling relations emerge and which includes the effects of gravity. Our main conclusions are as follows:

(i) a simple SF model of rising damp predicts values of the steady height of rise that are consistent with field observations,
(ii) the dynamics depends on the dimensionless quantity \( \alpha \), so that the primary factors are the wall thickness, the sorptivity of the wall materials and the potential evaporation of the immediate microenvironment,
(iii) although gravity limits the ultimate height of rise and retards the approach to capillary rise steady state, gravity effects can normally be neglected as capillary forces are dominant, and
(iv) the SF model provides a rational basis for understanding field observations and also for designing optimal conservation treatments.

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Appendix A

We set out here the full theory to include the effect of the gravitational force acting on the water in the wall. We denote by \( h_\infty \) the ultimate height to which moisture would rise in the complete absence of evaporation. While \( h_\infty \) may seem to be a quantity about which we can know little, this is not so, since at capillary rise equilibrium, the capillary pressure potential \( \Psi \) and the gravitational potential are equal at every height (Gummerson et al. 1980). For many constructional materials, the capillary potential \( \Psi \), as a function of the water content \( \theta \), is known from laboratory measurements using pressure plate and other methods (Hall & Hoff 2002). From this, we can compute the entire distribution of water content \( \theta \) with height \( z \), and we may reasonably estimate \( h_\infty \) as the SF equivalent height which contains the same total amount of water. Thus

\[
h_\infty = \frac{1}{\theta_s} \int_0^{\theta_s} \Psi \ d\theta,
\]

where \( \theta_s \) is the water content at saturation.

To include gravity effects, we use the SF equation for the approach to gravitational capillary rise equilibrium (Gummerson et al. 1980; Hall & Hoff 2002),
so that we have

\[ U = \frac{bS^2}{2\theta_w h} \left( 1 - \frac{h}{h_\infty} \right). \]  

(A 2)

This reduces to equation (3.2) when \( h/h_\infty \to 0 \). Then, with \( Q = \theta_w bh \), \( dQ/dt = U - E \) and \( E = \epsilon h \), we obtain, for the capillary rise dynamics with both evaporation and gravity, the differential equation

\[ \frac{d}{dt} h = \frac{bS^2}{2\theta_w h} \left( 1 - \frac{h}{h_\infty} \right). \]  

(A 3)

The mathematics can be simplified by using the dimensionless variables \( H = h/h_\infty \) and \( T = S^2 t/(2\theta_w h_\infty^2) \), so that equation (A 3) becomes

\[ \frac{dH}{dT} = \frac{1-H}{H} - \alpha H, \]  

(A 4)

where \( \alpha = 2\epsilon \theta_w h_\infty^2/bS^2 \). We see also that \( \alpha = (a/c)h_\infty^2 \), where \( a \) and \( c \) are the parameters defined in equation (3.7) for the no-gravity case.

The complete solution of this differential equation for the initial condition \( H = 0, \ T = 0 \) is

\[ -T = \frac{1}{2\alpha} \ln |\alpha H^2 + H - 1| + \frac{1}{2\alpha \beta} \ln \left| \frac{(2\alpha H + 1 + \beta)(1 - \beta)}{(2\alpha H + 1 - \beta)(1 + \beta)} \right|, \]  

(A 5)

where \( \beta = (1 + 4\alpha)^{1/2} \).

Equation (A 5) describes the entire process of capillary rise. The single parameter \( \alpha \) is a dimensionless group that can, however, take a wide range of values. The steady-state height of capillary rise \( H_{ss} \) obtained from equation (A 4) when \( dH/dT = 0 \) and \( T \to \infty \) is

\[ H_{ss} = \frac{h_{ss}}{h_\infty} = \frac{\beta - 1}{2\alpha}. \]  

(A 6)

For \( \alpha \gg 1 \), we can set \( \beta = 2\alpha^{1/2} \), and substituting for \( \alpha, H \) and \( T \), we recover the simpler equation (3.7). This is the case where gravity effects are negligible, and occurs when the capillary forces are strong, so that \( h_\infty \) is large and the evaporation rate \( e \) is appreciable. At long times, the system reaches a capillary rise steady state \( H_{ss} \), with a finite flow rate \( F_{ss} \). We obtain the result \( H_{ss} = \alpha^{-1/2} \) and \( h_{ss} \to S(b/2e \theta_w)^{1/2} \), as obtained previously in equation (3.4). From an initial dry state, the steady state is reached in a time \( t \approx 3b\theta_w/2e \).

In the other limit, if \( \alpha \ll 1 \), equation (A 5) reduces to the expression we have given elsewhere (Hall & Hoff 2002) for simple capillary rise equilibrium. This case is most easily obtained by switching off the evaporation, so that \( e = 0 \). At long times, the system reaches a true equilibrium \( H(\infty) = 1 \), where the flow rate \( F(\infty) = 0 \).

For the general case, with intermediate values of \( \alpha \), the full equations (A 5) and (A 6) must be used. At long times, the system reaches a steady state \( H_{ss} = (\beta - 1)/2\alpha \), with finite flow rate \( F_{ss} = e h_{ss} H_{ss} \).
In figure 3, the influence of the parameter $\alpha$ on the capillary rise dynamics is shown graphically.

References


